# DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY,
## LONERE – RAIGAD 402 103
## Summer End Semester Examination –2022

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**Branch: B. Tech. (Common to all)** **Semester: II**

**Subject with Subject Code: Engineering Mathematics – II (BTBS 201)** **Marks: 60**

**Date: 17/08/2022** **Time: 3.45 Hrs.**

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**Instructions to the Students**

1. Illustrate your answers with neat sketches, diagrams, etc., wherever necessary.
2. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

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**Q. 1**

(a) If the sum and product of two complex numbers are real, show that those two numbers must be either real or conjugate**.** [4 *Marks*]

(b) Solve the equation $\boldsymbol{x^6 - i = 0}$. [4 *Marks*]

(c) If $\boldsymbol{\tan(A + iB) = x + iy}$, prove that

(i) $\boldsymbol{\tan 2A = \frac{2x}{1-x^2-y^2}}$ (ii) $\boldsymbol{\tanh 2B = \frac{2y}{1+x^2+y^2}}$. [4 *Marks*]

**Q. 2**

(a) Solve: $\boldsymbol{\cos^2 x \frac{dy}{dx} + y = \tan x}$. [4 *Marks*]

(b) Solve: $\boldsymbol{(x^2 + y^2)dx - xy\, dy = 0}$. [4 *Marks*]

(c) Solve: $\boldsymbol{\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y.}$

[4 *Marks*]

**Q. 3 Solve any THREE:**

(a) Solve $\boldsymbol{(D^6 - D^4)y = x^2}$. [4 *Marks*]

(b) Solve $\boldsymbol{(D^2 - 2D + 1)y = x\, e^x \cos x}$. [4 *Marks*]

(c) Solve by the method of variation of parameters: $\boldsymbol{\frac{d^2y}{dx^2} + y = \operatorname{cosec} x}$. [4 *Marks*]

(d) Solve: $\boldsymbol{x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)}$. [4 *Marks*]

**Q. 4 Solve any TWO:**

(a) Find the Fourier series of the function $\boldsymbol{f(x) = x}$ in the interval $\boldsymbol{(0, 2\pi)}$. [6 *Marks*]

(b) Find the Fourier series expansion for the function $\boldsymbol{f(x) = x - x^2}$ in $\boldsymbol{-1 < x < 1}$. [6 *Marks*]

(c) Expand the function $\boldsymbol{f(x) = \pi x - x^2}$ in a half-range sine series in the interval $\boldsymbol{(0, \pi)}$. [6 *Marks*]

**Q. 5 Solve any THREE**

(a) Find $\boldsymbol{\nabla \cdot \vec{F}}$, where $\boldsymbol{\vec{F} = \left(\frac{x}{r}\right)\hat{i} + \left(\frac{y}{r}\right)\hat{j} + \left(\frac{z}{r}\right)\hat{k}}$. [4 *Marks*]

(b) Find **curl** $\boldsymbol{\vec{F}}$, where $\boldsymbol{\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)}$. [4 *Marks*]

(c) If $\vec{r}$ is a position vector with $\boldsymbol{r = |\vec{r}|}$, show that

$\boldsymbol{\nabla^2 r^n = n(n+1)r^{n-2}}$. [4 *Marks*]

(d) Verify the Green's theorem for $\boldsymbol{\int_C \{(xy + y^2)dx + x^2 dy\}}$

where $C$ is bounded by $\boldsymbol{y = x}$ and $\boldsymbol{y = x^2}$. [4 *Marks*]

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